CHAPTER 22

50/60Hz Transformers: Single and Three Phase

A consequence of power electronics in conjunction with any transformer is the possibility of dc voltage or current components imposed on the transformer. A dc current component biases the flux operating point, whilst a dc voltage component will lead to core saturation as the voltage-time integral creates an increasing flux.

22.1 DC *MMF*s in converter transformers

Half-wave rectification – whether controlled, semi-controlled or uncontrolled, is notorious for producing a dc *mmf* in transformers and triplen harmonics in the ac supply neutral of three-phase circuits. Generally, a transformer based solution can minimise the problem. In order to simplify the underlying concepts, a constant dc load current I_o is assumed, that is, the load inductance is assumed infinite. The transformer is assumed linear, no-load excitation is ignored, and the ac supply is assumed sinusoidal.

Independent of the transformer and its winding connection, the average output voltage from a rectifier, when the rectifier bridge input rms voltage is V_B and there are q pulses in the output, is given by

$$V_o = \frac{\hat{V}_B}{2\pi / q}\int_{-\pi/q}^{\pi/q} \cos\omega t \, d\omega t = \hat{V}_B \frac{\sin \pi / q}{\pi / q} \qquad (22.1)$$

The rectifier bridge rms voltage output is dominated by the dc component and is given by

$$V_{o\,rms} = \frac{q}{2\pi}\int_{-\frac{\pi}{q}}^{+\frac{\pi}{q}} 2V_B^2 \cos^2(\omega t)\, d\omega t = V_B\sqrt{1+\frac{q}{2\pi}\sin\frac{2\pi}{q}} \qquad (22.2)$$

The Fourier expression for the output voltage, which is also dominated by the dc component, is

$$v_o(\omega t) = V_o + V_o\sum_{k=1}^{\infty}\frac{2(-1)^{k+1}}{k^2n^2-1}\cos kn\omega t \qquad (22.3)$$

22.1.1 Effect of multiple coils on multiple limb transformers

The transformer for a single-phase two-pulse half-wave rectifier has three windings, a primary and two secondary windings as shown in figure 22.1. Two possible transformer core and winding configurations are shown, namely shell and core. In each case the winding turns ratios are identical, as is the load voltage and current, but the physical transformer limb arrangements are different. One transformer, figure 22.1a, has three limbs (made up from E and I laminations), while the second, figure 22.1b, is made from a circular core (shown as a square core). The reason for the two possibilities is related to the fact that the circular core can use a single strip of wound cold-rolled grain-orientated silicon steel as lamination material. Such steels offer better magnetic properties than the non-oriented steel that must be used for E core laminations. Single-phase toroidal core transformers are attractive because of the reduced size and weight but manufacturers do not highlight their inherent limitation and susceptibility to dc flux biasing, particularly in half-wave type applications. Although the solution is simple, the advantageous features of the toroidal transformer are lost, as will be shown.

i. The E-I three-limb transformer (shell)

The key feature of the three-limb shell is that the three windings are on the centre limb, as shown in figure 22.1a. The area of each outer limb is half that of the central limb. Assuming a constant load current I_o and equal secondary turns, N_s, excitation of only the central limb yields the following *mmf* equation

$$mmf = i_pN_p + i_{s1}N_s - i_{s2}N_s \qquad (22.4)$$

Thus the primary current i_p is

$$i_p = \frac{N_s}{N_p}(i_{s2} - i_{s1}) + \frac{mmf}{N_p} \qquad (22.5)$$

From the waveforms in figure 22.1a, since i_{s2} - i_{s1} is alternating, an average primary current of zero in equation (22.5) can only be satisfied by $mmf = 0$.

The various transformer voltages and currents are

$$I_{s1} = I_{s2} = I_s = \frac{I_o}{\sqrt{2}}$$
$$I_p = I_o\frac{N_s}{N_p} \qquad (22.6)$$
$$V_{s1} = V_{s2} = V_s = V_p\frac{N_s}{N_p} = \frac{\pi}{2\sqrt{2}}V_o$$

Therefore the transformer input, output and average VA ratings are

$$S_s = V_{s1}I_{s1} + V_{s2}I_{s2} = \sqrt{2}\frac{N_s}{N_p}V_pI_o\left(=\frac{\pi}{2}P_o = 1.57P_o\right)$$
$$S_p = V_pI_p = \frac{N_s}{N_p}V_pI_o\left(=\frac{\pi}{2\sqrt{2}}P_o = 1.11P_o\right) \qquad (22.7)$$
$$\overline{S} = ½(S_s + S_p) = \frac{N_p}{N_s}V_pI_o\frac{1+\sqrt{2}}{2}$$

The average output voltage, hence output power, are

$$V_o = \frac{2\sqrt{2}}{\pi}V_s = \frac{2\sqrt{2}}{\pi}\frac{N_s}{N_p}V_p = 0.9\frac{N_s}{N_p}V_p \qquad (22.8)$$
$$P_o = I_oV_o$$

Thus

$$\overline{S} = \frac{1+\sqrt{2}}{4\sqrt{2}}\pi P_o = 1.34P_o \qquad (22.9)$$

Since the transformer primary current is the line current, the supply power factor is

$$pf = \frac{P_o}{\overline{S}} = \frac{V_oI_o}{V_pI_p} = \frac{\frac{2}{\pi}V_s\,I_o}{\frac{N_p}{N_s}V_s\,\frac{N_s}{N_p}I_o} = \frac{2\sqrt{2}}{\pi} = 0.9 \qquad (22.10)$$

ii. The two-limb strip core transformer

Figure 22.1b shows the windings equally split on each transformer leg. In practice the windings can all be on one leg and the primary is one coil, but separation as shown allows visual *mmf* analysis. The load and diode currents and voltages are the same as for the E-I core arrangement, as seen in the waveforms in figure 22.1b. The *mmf* analysis necessary to assess the primary currents and core flux, is based on analysing each limb.

$$mmf_1 = -i_p½N_p + i_{s1}N_s$$
$$mmf_2 = +i_p½N_p + i_{s2}N_s \qquad (22.11)$$
$$mmf_1 = mmf_2 = mmf$$

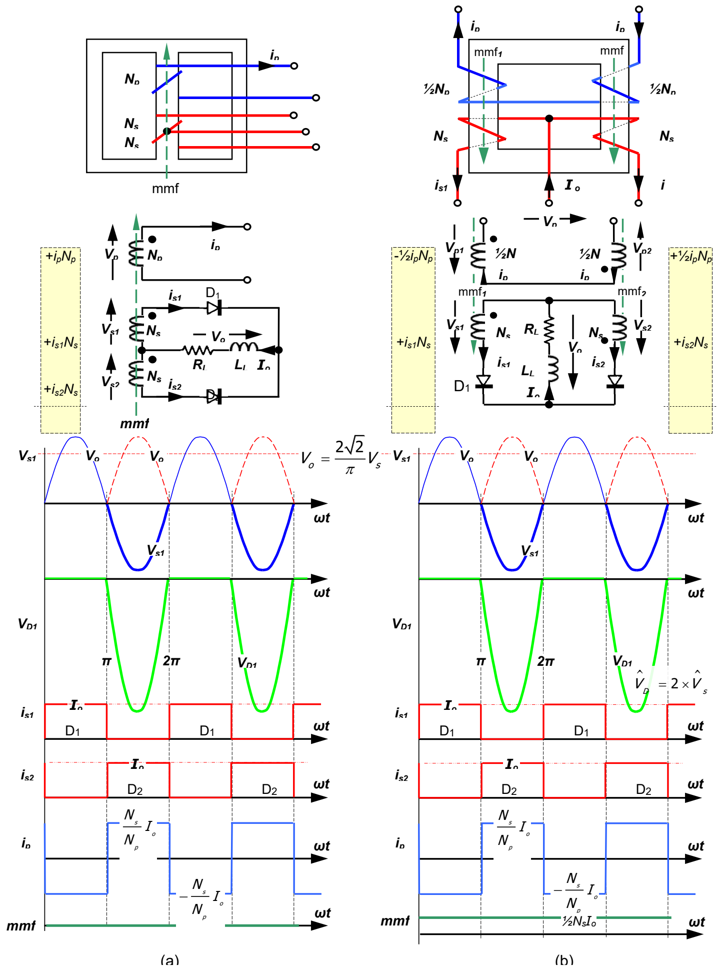


Figure 22.1. *Single-phase transformer core and winding arrangements:*
(a) E-I core with zero dc mmf bias and (b) square/circular core with dc mmf bias.

These equations yield

$$mmf = N_s \tfrac{1}{2}\left(i_{s1} + i_{s2}\right) = N_s \tfrac{1}{2} I_o$$
$$i_p = \frac{N_s}{N_p}\left(i_{s1} - i_{s2}\right) \qquad (22.12)$$

These two equations are used every ac half cycle to obtain the plots in figure 22.1b. It will be noticed that the core has a magnetic *mmf* bias of ½$N_s I_o$ associated with the half-wave rectification process.

The various transformer ratings are

$$I_{s1} = I_{s2} = I_s = \frac{I_o}{\sqrt{2}} \qquad I_p = I_o \frac{N_s}{N_p}$$
$$V_{p1} = V_{p2} = \tfrac{1}{2} V_p \qquad V_{s1} = V_{s2} = V_s = \tfrac{1}{2} V_p \frac{N_s}{\tfrac{1}{2} N_p} = V_p \frac{N_s}{N_p} \qquad (22.13)$$
$$V_p = \frac{N_p}{N_s} \frac{\pi}{2\sqrt{2}} V_o$$

Therefore the transformer VA ratings are

$$S_s = V_{s1} I_{s1} + V_{s2} I_{s2} = \sqrt{2} \frac{N_s}{N_p} V_p I_o \qquad \left(= \frac{\pi}{2} P_o = 1.57 P_o\right)$$
$$S_p = V_{p1} I_p + V_{p2} I_p = \frac{N_s}{N_p} V_p I_o \qquad \left(= \frac{\pi}{2\sqrt{2}} P_o = 1.11 P_o\right) \qquad (22.14)$$
$$\bar{S} = \tfrac{1}{2}\left(S_s + S_p\right) = \frac{1+\sqrt{2}}{2} \frac{N_s}{N_p} V_p I_o$$

The average output voltage, hence output power, are

$$V_o = \frac{2\sqrt{2}}{\pi} V_s = \frac{2\sqrt{2}}{\pi} \frac{N_s}{\tfrac{1}{2} N_p} \tfrac{1}{2} V_p = \frac{2\sqrt{2}}{\pi} \frac{N_s}{N_p} V_p \qquad (22.15)$$
$$P_o = I_o V_o$$

Thus

$$\bar{S} = \pi \frac{1+\sqrt{2}}{4\sqrt{2}} P_o = 1.34 P_o \qquad (22.16)$$

and the supply power factor is $pf = P_o / \bar{S} = 0.9$.

The interpretation for equations (22.14) and (22.16) (and equations (22.7) and (22.9)) is that the transformer has to be oversized by 11% on the primary side and 57% on the secondary. From equation (22.16), in terms of the average VA, the transformer needs to be 34% larger than that implied by the rated dc load power. Further, the secondary is rated higher than the primary because of a dc component in the secondary. This core saturation aspect requires special attention when dimensioning the core size. Additionally, a component of the over rating requirement is due to circulating harmonics that do not contribute to real power output. This component is particularly relevant in three-phase delta primary or secondary connections when co-phasal triplens circulate. This discussion on apparent power aspects is relevant to all the transformer connections considered. Generally the higher the phase number the better the transformer core utilisation, but the poorer the secondary winding and rectifying diode utilisation since the percentage current conduction decreases with increased pulse number.

The fundamental ripple in the output voltage, at twice the supply frequency, is ⅔V_o.

The two cores give the same rated transformer apparent power and supply power factor, but importantly, undesirably, the toroidal core suffers an *mmf* magnetic bias.

In each core case each diode conducts for 180º and

$$\bar{I}_D = \tfrac{1}{2} I_o \qquad I_{D\,rms} = \frac{I_o}{\sqrt{2}} \qquad \hat{V}_D = 2\sqrt{2} \frac{N_s}{N_p} V_p \qquad (22.17)$$

With a purely resistive load, a full-wave rectifier with a centre-tapped primary gives

$$\bar{I}_D = \tfrac{1}{2} I_o \qquad I_{D\,rms} = \tfrac{1}{4}\pi I_o \qquad S_s = 1.75 P_o \qquad S_p = 1.23 P_o \qquad \bar{S} = 1.49 P_o \qquad (22.18)$$

22.1.2 Single-phase toroidal core mmf imbalance cancellation – zig-zag winding

In figure 22.2, each limb of the core has an extra secondary winding, of the same number of turns, N_s. *MMF* analysis of each limb in figure 22.2 yields

$$\begin{aligned} \text{limb 1:} \quad & mmf_o = -i_p N_p - i_{s2} N_s + i_{s1} N_s \\ \text{limb 2:} \quad & mmf_o = i_p N_p + i_{s2} N_s - i_{s1} N_s \end{aligned} \qquad (22.19)$$

Adding the two mmf equations gives $mmf_o = 0$ and the resulting alternating primary current is given by

$$i_p = \frac{N_s}{N_p}\left(i_{s1} - i_{s2}\right) \qquad (22.20)$$

The transformer apparent and real power are rated by the same equation as for the previous winding arrangements, namely

$$\bar{S} = \tfrac{1}{2}\left(S_p + S_s\right) = \tfrac{1}{2}\left(\frac{\pi}{2\sqrt{2}}P_o + \frac{\pi}{2}P_o\right) = 1.34P_o$$
$$\text{where } P_o = V_o I_o \text{ and } V_o = \frac{N_s}{N_p}\frac{2\sqrt{2}}{\pi}V_p \qquad (22.21)$$

Since the transformer primary current is the ac line current, the supply power factor is $pf = P_o / \bar{S} = 0.9$.

The general rule to avoid any core dc *mmf* is, each core leg must be effectively excited by a net alternating current.

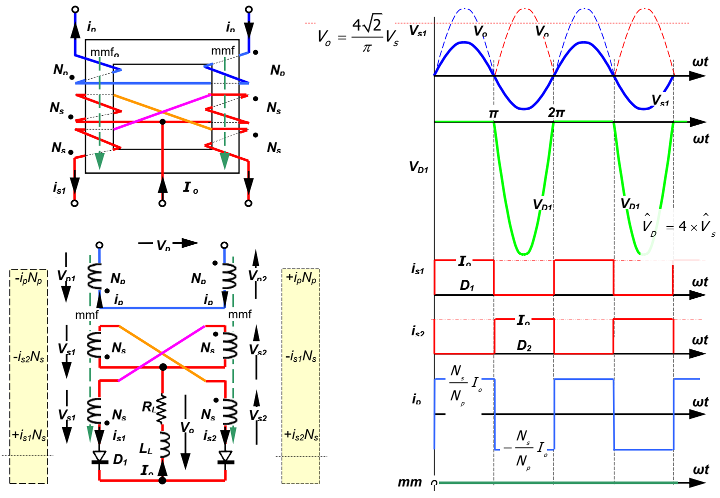


Figure 22.2. *Single-phase zig-zag transformer core and winding arrangement using square/circular core with zero dc mmf bias.*

22.1.3 Single-phase transformer connection, with full-wave rectification

The secondary current is ac with a zero average , thus no core mmf bias occurs.
The average output voltage and peak diode reverse voltage, in terms of the transformer secondary rms voltage, are

$$V_o = \frac{2\sqrt{2}}{\pi}V_s \qquad V_{Dr} = \sqrt{2}V_s \qquad (22.22)$$

The rms output voltage is the bridge input rms voltage:

$$V_{o\,rms} = V_s \qquad (22.23)$$

The various harmonic currents are

$$I_{s1} = \frac{2\sqrt{2}}{\pi}I_o = 0.9I_o \qquad I_{sh} = \frac{I_{s1}}{h} \quad \text{for } h \text{ odd} \qquad (22.24)$$

The power factor angle of the fundamental is unity, while the THD is 48.43%.

The transformer primary and secondary apparent powers are

$$S_p = S_s = \frac{\pi}{2\sqrt{2}}P_o = 1.11P_o \qquad (22.25)$$

The transformer average VAr rating is

$$\bar{S} = \frac{\pi}{2\sqrt{2}}P_o \qquad (22.26)$$

Since the line current is the primary current, the supply power factor is

$$pf = \frac{P_o}{\bar{S}} = \frac{2\sqrt{2}}{\pi} \qquad (22.27)$$

The fundamental ripple in the output voltage, at twice the supply frequency, is ⅔V_o.

With a purely resistive load (as opposed to a constant load current), a full-wave bridge rectifier gives

$$I_o = \frac{V_o}{R} \qquad I_{o\,rms} = \frac{V_s}{R}$$
$$\bar{I}_D = \tfrac{1}{2}I_o \qquad I_{D\,rms} = \tfrac{1}{4}\pi I_o \qquad S_s = 1.23P_o \qquad S_p = 1.23P_o \qquad \bar{S} = 1.23P_o \qquad (22.28)$$

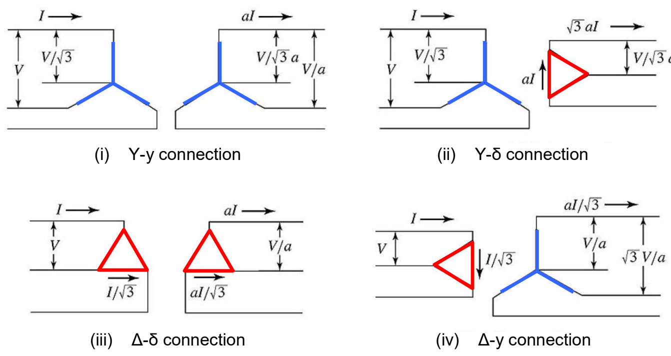


Figure 22.3. Four connections for a three-phase transformer.

22.1.4 Three-phase transformer connections

Basic three-phase transformers can have a combination of star (wye) and delta, primary and secondary winding arrangements, figure 22.3. For a given line voltage and current, a wye connection reduces the phase winding voltage by √3, while a delta configuration reduces the phase winding current by √3.

i. Y - y (WYE-wye) is avoided due to imbalance and third harmonic problems, but with an extra delta (tertiary) winding, triplen problems can be minimised. The arrangement is used to interconnect high voltage networks, 240kV/345kV or when two neutrals are needed for grounding.

ii. Y - δ (WYE-delta) is commonly used for (high voltage) step-down voltage applications.

iii. Δ – δ (DELTA-delta) is used in 11kV medium voltage applications where neither primary nor neutral connection is needed, for example some industrial applications. LV large currents.

iv. Δ - y (DELTA-wye) is used as a step-up transformer at the point of generation, before transmission and industrial/commercial applications. Also in three phase 4 wire systems.

The open delta transformer connection can be made with only two single-phase transformers. This connection can be used when the three-phase power requirement is not excessive. The output power of an open delta connection is only 87.7% of the rated power of the two transformers.

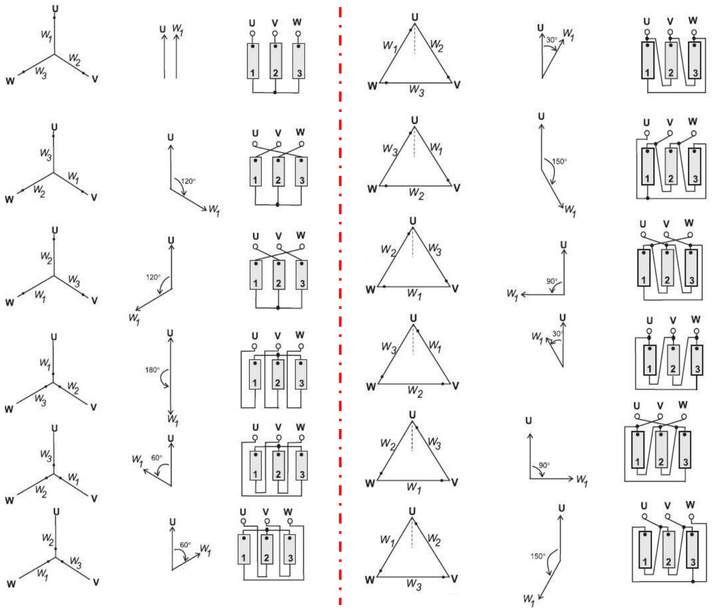


Figure 22.4. Six ways to connect a wye (left) and delta (right) winding.

Independent of the three-phase connection of the primary and secondary (six possibilities shown in figure 22.4), for a balance three-phase load, the apparent power, VA, from the supply to the load is

$$S = \sqrt{3} V_{line} I_{line} = 3 V_{phase} I_{phase} \qquad (22.29)$$

Also the sum of the primary and secondary line voltages is zero, that is

$$\begin{aligned} V_{AB} + V_{BC} + V_{CA} &= 0 \\ V_{ab} + V_{bc} + V_{ca} &= 0 \end{aligned} \qquad (22.30)$$

where upper case subscripts refer to the primary and lower case subscripts refer to the secondary.

Y-y *(WYE-wye) connection*

Electrically, the *Y-y* transformer connection shown in figure 22.5, can be summarized as follows.

$$\eta_{Y-y} = \frac{N_p}{N_s} = \frac{V_{AN}}{V_{an}} = \frac{I_a}{I_A} = \frac{I_a}{I_{L1}} = \frac{V_{BN}}{V_{bn}} = \frac{I_b}{I_B} = \frac{V_{CN}}{V_{cn}} = \frac{I_c}{I_C} \qquad (22.31)$$

$$\begin{aligned} &V_{AB} = V_{AN} + V_{NB} = V_{AN} - V_{BN} = \sqrt{3} V_{AN} e^{j30^\circ} = \sqrt{3} V_{AN} \angle 30^\circ \\ &V_{BC} = V_{BN} + V_{NC} = V_{BN} - V_{CN} \quad V_{CA} = V_{CN} + V_{NA} = V_{CN} - V_{AN} \\ &V_{ab} = V_{an} - V_{bn} = \sqrt{3} V_{an} e^{j30^\circ} = \sqrt{3} V_{an} \angle 30^\circ \\ &V_{bc} = V_{bn} - V_{cn} \quad V_{ca} = V_{cn} - V_{an} \end{aligned} \qquad (22.32)$$

$$I_N = I_A + I_B + I_C \qquad I_n = I_a + I_b + I_c$$

The output current rating is (22.33)

$$I_Y = \frac{|S|/3}{V/\sqrt{3}} = \frac{|S|}{\sqrt{3}V}$$

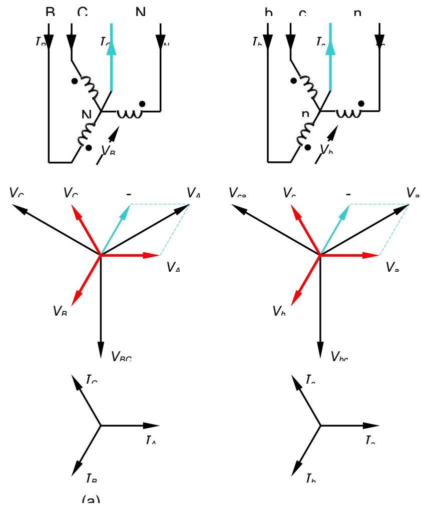


Figure 22.5. *Three-phase Y-y transformer:*
(a) winding arrangement and (b) phasor diagrams.

Y- δ *(WYE-delta) connection*

The *Y*- δ transformer connection in figure 22.6 can be summarized as follows.

$$\eta_{Y-\delta} = \frac{N_p}{N_s} = \frac{V_{AN}}{V_{ab}} = \frac{I_{ba}}{I_A} = \frac{V_{BN}}{V_{bc}} = \frac{I_{cb}}{I_B} = \frac{V_{CN}}{V_{ca}} = \frac{I_{ac}}{I_C} \quad (22.34)$$

$$V_{AB} = V_{AN} - V_{BN} = V_{AN} - V_{AN}e^{-j120^\circ} = \sqrt{3}V_{AN}e^{j30^\circ}$$

$$I_a = I_{ba} - I_{ac} = I_{ba} - I_{ba}e^{-j240^\circ} = \sqrt{3}\,I_{ba}\,e^{-j30^\circ}$$

$$I_a + I_b + I_c = 0$$

The output current rating is (22.35)

$$I_\Delta = \frac{|S|/3}{V} = \frac{|S|}{3V}$$

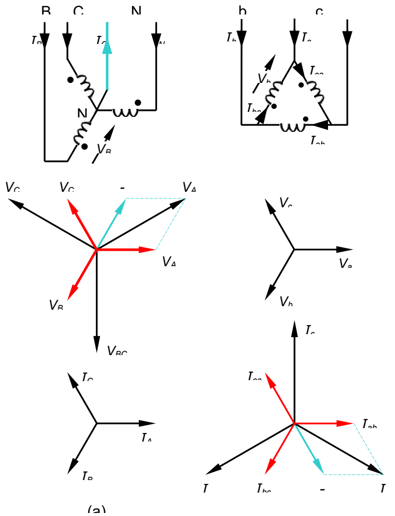


Figure 22.6. *Three-phase Y- δ transformer:*
(a) winding arrangement and (b) phasor diagrams.

Δ-δ (DELTA-delta) connection

In figure 22.7, the Δ - δ transformer connection can be summarized as follows.

$$\eta_{\Delta-\delta} = \frac{N_p}{N_s} = \frac{V_{AB}}{V_{ab}} = \frac{I_a}{I_A} = \frac{I_{ba}}{I_{AB}} = \frac{V_{BC}}{V_{bc}} = \frac{I_b}{I_B} = \frac{I_{cb}}{I_{BC}} = \frac{V_{CA}}{V_{ca}} = \frac{I_c}{I_C} = \frac{I_{cb}}{I_{CA}} \quad (22.36)$$

$$I_A = I_{AB} - I_{CA} = \sqrt{3}\,I_{AB}e^{-j30^\circ} = \sqrt{3}I_{AB}\angle -30^\circ$$

$$I_B = I_{BC} - I_{AB} \qquad I_C = I_{CA} - I_{BC}$$

$$I_A + I_B + I_C = 0 \quad (22.37)$$

$$I_a = I_{ab} - I_{ca} = \sqrt{3}\,I_{ab}e^{-j30^\circ} = \sqrt{3}I_{ab}\angle -30^\circ$$

$$I_b = I_{cb} - I_{ba} \qquad I_c = I_{ac} - I_{cb}$$

$$I_c + I_b + I_c = 0$$

The output current rating is

$$I_Y = \frac{|S|/3}{V/\sqrt{3}} = \frac{|S|}{\sqrt{3}V} \quad (22.38)$$

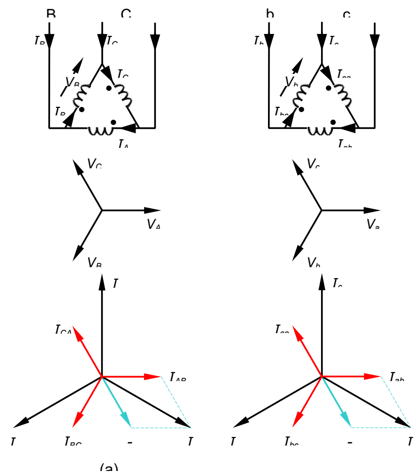


Figure 22.7. *Three-phase Δ-δ transformer:*
(a) winding arrangement and (b) phasor diagrams.

Δ-y (DELTA-wye) connection

The Δ-*y* transformer connection in figure 22.8 can be summarized as follows.

$$\eta_{\Delta-y} = \frac{V_{AB}}{V_{ab}} = \frac{V_{AB}}{V_{ab}}\frac{e^{-j30^\circ}}{\sqrt{3}} = \frac{V_{AN}}{V_{an}} = \frac{I_a^*}{I_A^*} = \frac{I_a^*}{\left(\sqrt{3}\,I_{AB}e^{-j30^\circ}\right)^*} \tag{22.39}$$

$$I_A = I_{AB} - I_{CA} = I_{AB} - I_{AB}e^{-j240^\circ} = \sqrt{3}\,I_{an}e^{-j30^\circ}$$

The output current rating is

$$I_Y = \frac{|S|/3}{V/\sqrt{3}} = \frac{|S|}{\sqrt{3}V} \tag{22.40}$$

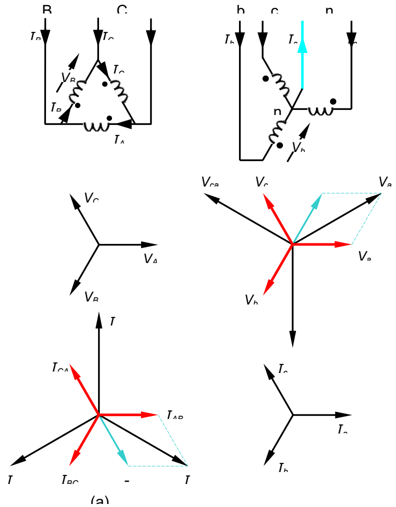


Figure 22.8. *Three-phase Δ-y transformer: (a) winding arrangement and (b) phasor diagrams.*

22.1.5 Three-phase transformer, half-wave rectifiers - core mmf imbalance

A delta secondary connection cannot be used for half-wave rectification since no physical neutral connection exists.

i. Star connected primary Y-y (WYE-wye)

The three-phase half-wave rectifier with a star-star connected transformer in figure 22.9a is prone to magnetic *mmf* core bias. With a constant load current I_o, each diode conducts for 120º. Each leg is analysed on an *mmf* basis, and the current and *mmf* waveforms in figure 22.9a are derived as follows.

$$\begin{aligned} mmf_o &= N_s i_{s1} - N_p i_{p1} \\ mmf_o &= N_s i_{s2} - N_p i_{p2} \\ mmf_o &= N_s i_{s3} - N_p i_{p3} \end{aligned} \tag{22.41}$$

By symmetry and balance, the *mmf* in each leg must be equal.

If i_N is the neutral current then the equation for the currents is

$$i_{p1} + i_{p2} + i_{p3} = i_N \tag{22.42}$$

The same *mmf* equations are obtained if the load is purely resistive.
Any triplens in the primary will add algebraically, while any other harmonics will vectorially cancel to zero. Therefore the neutral may only conduct primary side triplen currents. Any input current harmonics are due to the rectifier and the rectifier harmonics of the order $h = cp \pm 1$ where $c = 0,1,2,\ldots$ and p is the pulse number, 3. No secondary-side third harmonics can exist hence $h \neq 3k$ for $k = 1, 2, 3,$. Therefore no primary-side triplen harmonic currents exist to flow in the neutral, that is $i_N = 0$. In a balanced load condition, the neutral connection is redundant. The system equations resolve to

$$\begin{aligned} i_{p1} &= \frac{N_s}{N_p}\left(\tfrac{2}{3}i_{s1} - \tfrac{1}{3}i_{s2} - \tfrac{1}{3}i_{s3}\right) \\ i_{p2} &= \frac{N_s}{N_p}\left(-\tfrac{1}{3}i_{s1} + \tfrac{2}{3}i_{s2} - \tfrac{1}{3}i_{s3}\right) \\ i_{p3} &= \frac{N_s}{N_p}\left(-\tfrac{1}{3}i_{s1} - \tfrac{1}{3}i_{s2} + \tfrac{2}{3}i_{s3}\right) \\ mmf_o &= N_s\left(\frac{i_{s1} + i_{s2} + i_{s3}}{3}\right) = \tfrac{1}{3}N_s I_o \end{aligned} \tag{22.43}$$

Specifically, the core has an mmf dc bias of $\tfrac{1}{3}N_sI_o$.

Waveforms satisfying these equations are show plotted in figure 22.9a. The various transformer currents and voltages are

$$\begin{aligned} I_{s1} &= I_{s2} = I_{s3} = I_s = \frac{I_o}{\sqrt{3}} \\ I_{p1} &= I_{p2} = I_{p3} = I_p = \frac{\sqrt{2}}{3}\frac{N_s}{N_p}I_o \\ V_{p1} &= V_{p2} = V_{p3} = V_p = V_o\frac{N_p}{N_s}\frac{2\pi}{3\sqrt{6}} \\ V_{s1} &= V_{s2} = V_{s3} = V_s = V_o\frac{2\pi}{3\sqrt{6}} = \frac{V_o}{1.17} \end{aligned} \tag{22.44}$$

The fundamental ripple in the output voltage, at three times the supply frequency, is ¼Vₒ.
The various transformer VA ratings are

$$\begin{aligned} S_s &= V_{s1}I_{s1} + V_{s2}I_{s2} + V_{s3}I_{s3} = 3V_sI_s = \frac{\pi\sqrt{2}}{3}P_o = 1.48P_o \\ S_p &= V_{p1}I_p + V_{p2}I_p + V_{p3}I_p = 3V_pI_p = \frac{2}{3\sqrt{3}}P_o = 1.21P_o \\ \overline{S} &= ½\left(S_s + S_p\right) = P_o\frac{2 + \pi\sqrt{6}}{3\sqrt{3}} = 1.34P_o \end{aligned} \tag{22.45}$$

The average output power is

$$P_o = I_oV_o \tag{22.46}$$

Since with a wye connected transformer primary, the transformer primary phase current is the line current, the supply power factor is

$$pf = \frac{P_o}{\overline{S}} = \frac{V_oI_o}{3V_pI_p} = \frac{\frac{3}{\pi}\sqrt{2}V_s\frac{\sqrt{3}}{2}I_o}{3\frac{N_p}{N_s}V_s\frac{\sqrt{3}}{2}\frac{N_s}{N_p}I_o} = 0.827 \tag{22.47}$$

Although the neutral connection is redundant for a constant load current, the situation is different if the load current has ripple at the three times the rectified ac frequency, as with a resistive load. Equations in (22.43) remain valid for the untapped neutral case. In such a case, when triplens exist in the load current, how they are reflected into the primary depends on whether or not the neutral is connected:

- No neutral connection – a triplen *mmf* is superimposed on the *mmf* dc bias of $\tfrac{1}{3}N_sI_o$.
- Neutral connected – a dc current (zero sequence) flows in the neutral and the associated zero sequence line currents in the primary, oppose the generation of any triplen *mmf* onto the dc *mmf* bias of $\tfrac{1}{3}N_sI_o$.

ii. Delta connected primary Δ-y (DELTA-wye)

The three-phase half-wave rectifier with a delta-star connected transformer in figure 22.5b is prone to magnetic *mmf* core bias. With a constant load current I_o each diode conducts for 120º. Each leg is analysed on an *mmf* basis, and the current and *mmf* waveforms in figure 22.9b are derived as follows.

$$\begin{aligned} mmf_o &= N_s i_{s1} - N_p i_{p1} \\ mmf_o &= N_s i_{s2} - N_p i_{p2} \\ mmf_o &= N_s i_{s3} - N_p i_{p3} \end{aligned} \qquad (22.48)$$

$$i_{L1} = i_{p1} - i_{p3} \qquad i_{L2} = i_{p2} - i_{p1} \qquad i_{L3} = i_{p3} - i_{p2}$$

The line-side currents have average values of zero and if it is assumed that the core *mmf* has only a dc component, that is no alternating component, then based on these assumptions

$$mmf_o = N_s\left(\frac{i_{s1}+i_{s2}+i_{s3}}{3}\right) = \frac{1}{3}N_sI_o \qquad (22.49)$$

The primary currents are then

$$\begin{aligned} i_{p1} &= \left(i_{s1} - \frac{1}{3}I_o\right)\frac{N_s}{N_p} = \frac{N_s}{N_p}\left(\frac{2}{3}i_{s1} - \frac{1}{3}i_{s2} - \frac{1}{3}i_{s3}\right) \\ i_{p2} &= \left(i_{s2} - \frac{1}{3}I_o\right)\frac{N_s}{N_p} = \frac{N_s}{N_p}\left(-\frac{1}{3}i_{s1} + \frac{2}{3}i_{s2} - \frac{1}{3}i_{s3}\right) \\ i_{p3} &= \left(i_{s3} - \frac{1}{3}I_o\right)\frac{N_s}{N_p} = \frac{N_s}{N_p}\left(-\frac{1}{3}i_{s1} - \frac{1}{3}i_{s2} + \frac{2}{3}i_{s3}\right) \end{aligned} \qquad (22.50)$$

These line-side currents are the same as for the star connected primary, hence the same real and apparent power equations are also applicable to the delta connected primary transformer, viz. equations (22.45) and (22.46).

The line currents are

$$\begin{aligned} i_{L1} &= \frac{N_s}{N_p}\left(i_{p1} - i_{p3}\right) \\ i_{L2} &= \frac{N_s}{N_p}\left(i_{p2} - i_{p1}\right) \\ i_{L3} &= \frac{N_s}{N_p}\left(i_{p3} - i_{p2}\right) \end{aligned} \qquad (22.51)$$

The waveforms for these equations are shown plotted in figure 22.9b, where

$$I_p = \frac{N_s}{N_p}\frac{\sqrt{3}}{2}I_o \text{ and } I_L = \frac{N_s}{N_p}\sqrt{\frac{3}{2}}I_o \qquad (22.52)$$

that is $I_L = \sqrt{3}\,I_p$

$$\bar{S} = 1.34P_o \qquad (22.53)$$

The supply power factor is

$$pf = \frac{V_oI_o}{\sqrt{3}V_pI_L} == 0.827 \qquad (22.54)$$

Although with a delta connected primary, the ac supply line currents are not the transformer primary currents, the supply power factor is the same as a star primary connection since the proportions of the input harmonics are the same.

The rms output voltage is

$$V_{o\,rms} = \sqrt{2}V_s\sqrt{\frac{3}{2\pi}\left(\frac{\pi}{3} + \frac{\sqrt{3}}{4}\right)} \qquad (22.55)$$

Each diode conducts for 120º and

$$\bar{I}_D = \tfrac{1}{3}I_o \qquad I_{D\,rms} = \frac{I_o}{\sqrt{3}} \qquad V_D = \sqrt{6}\frac{N_s}{N_p}V_p \qquad (22.56)$$

The primary connection, delta or wye, does not influence any **dc** mmf generated in the core, although the primary connection does influence if an ***ac*** mmf results.

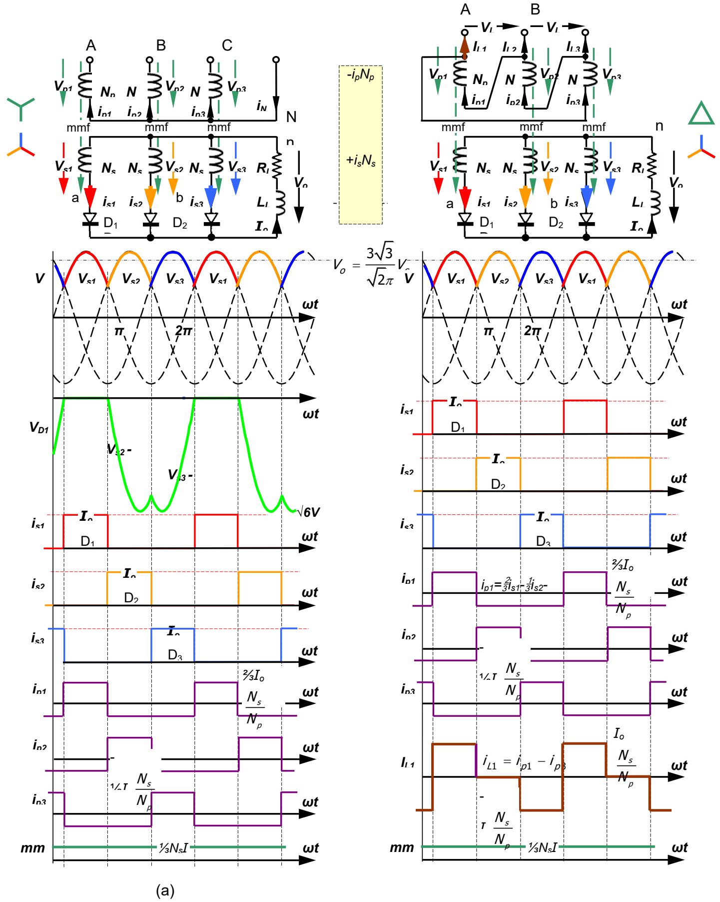


Figure 22.9. *Three-phase transformer winding arrangement with dc mmf bias: (a) star connected primary and (b) delta connected primary.*

22.1.6 Three-phase transformer with hexa-phase rectification, mmf imbalance

Figure 22.10 shown a tri-hexaphase half-wave rectifier, which can employ a wye or delta primary configuration, but only a star secondary connection is possible, since a neutral connection is required. The primary configuration can be shown to dictate core mmf bias conditions.

i. Y-y (WYE-wye) connection

The mmf balance for the wye primary connection in figure 22.10a is

$$\begin{aligned} N_s i_{s1} - N_s i_{s4} - N_p i_{p1} &= 0 \\ N_s i_{s3} - N_s i_{s6} - N_p i_{p2} &= 0 \\ N_s i_{s5} - N_s i_{s2} - N_p i_{p3} &= 0 \\ i_{p1} + i_{p2} + i_{p3} &= 0 \end{aligned} \qquad (22.57)$$

The primary currents expressed in terms of the secondary current are

$$\begin{aligned} i_{p1} &= \frac{N_s}{N_p}\left(\tfrac{2}{3} i_{s1} + \tfrac{1}{3} i_{s2} - \tfrac{1}{3} i_{s3} - \tfrac{2}{3} i_{s4} - \tfrac{1}{3} i_{s5} + \tfrac{1}{3} i_{s6}\right) \\ i_{p2} &= \frac{N_s}{N_p}\left(-\tfrac{1}{3} i_{s1} + \tfrac{1}{3} i_{s2} + \tfrac{2}{3} i_{s3} + \tfrac{1}{3} i_{s4} - \tfrac{1}{3} i_{s5} - \tfrac{2}{3} i_{s6}\right) \\ i_{p3} &= \frac{N_s}{N_p}\left(-\tfrac{1}{3} i_{s1} - \tfrac{2}{3} i_{s2} - \tfrac{1}{3} i_{s3} + \tfrac{1}{3} i_{s4} - \tfrac{2}{3} i_{s5} - \tfrac{1}{3} i_{s6}\right) \\ mmf &= N_s \frac{1}{3}\left(i_{s1} - i_{s2} + i_{s3} - i_{s4} + i_{s5} - i_{s6}\right) \end{aligned} \qquad (22.58)$$

These line side equations are plotted in figure 22.10a. Notice that an alternating mmf exists in the core related to the pulse frequency, $n = 2q = 6$.

The transformer primary currents and the line currents are

$$i_p = \frac{\sqrt{2}}{3} I_o \qquad i_L = \frac{\sqrt{2}}{3} I_o \qquad (22.59)$$

Note that because of the zero sequence current, triplens, in the delta primary that

$$i_L = \sqrt{2}\, i_p \quad \text{not} \quad i_L = \sqrt{3}\, i_p \qquad (22.60)$$

The transformer power ratings are

$$\begin{aligned} S_s &= 6\left(\frac{\pi}{3\sqrt{2}} V_o\right)\left(\frac{1}{\sqrt{6}} I_o\right) = \frac{\pi}{\sqrt{3}} P_o \\ S_p &= 3\left(\frac{\pi}{3\sqrt{2}} V_o\right)\left(\frac{\sqrt{2}}{3} I_o\right) = \frac{\pi}{3} P_o \\ \overline{S} &= ½\left(\frac{\pi}{\sqrt{3}} P_o + \frac{\pi}{3} P_o\right) = \frac{\pi}{6}\left(\sqrt{3} + 1\right) P_o = 1.43 P_o \end{aligned} \qquad (22.61)$$

ii. Δ-y (DELTA-wye) connection

When the primary is delta connected, as shown in figure 22.10b, the mmf equations are the same as with a wye primary, namely

$$\begin{aligned} N_s i_{s1} - N_s i_{s4} - N_p i_{p1} &= 0 \\ N_s i_{s3} - N_s i_{s6} - N_p i_{p2} &= 0 \\ N_s i_{s5} - N_s i_{s2} - N_p i_{p3} &= 0 \end{aligned} \qquad (22.62)$$

but Kirchhoff's electrical current equation becomes of the following form for each phase:

$$mmf = \frac{1}{2\pi} \int_0^{2\pi} N_s \left(i_{s1} - i_{s4}\right) d\omega t = 0 \qquad (22.63)$$

Thus since each limb experiences an alternating current, similar to i_{s1} - i_{s4} for each limb, with an average value of zero, the line currents can be calculated from

$$i_{p1} = \frac{N_s}{N_p}\left(i_{s1} - i_{s4}\right) \qquad i_{p2} = \frac{N_s}{N_p}\left(i_{s3} - i_{s6}\right) \qquad i_{p3} = \frac{N_s}{N_p}\left(i_{s5} - i_{s2}\right) \qquad (22.64)$$

The line currents are

$$\begin{aligned} i_{L1} &= i_{p1} - i_{p3} = \frac{N_s}{N_p}\left(i_{s1} + i_{s2} - i_{s4} - i_{s5}\right) \\ i_{L2} &= i_{p2} - i_{p1} = \frac{N_s}{N_p}\left(-i_{s1} + i_{s3} + i_{s4} - i_{s6}\right) \\ i_{L3} &= i_{p3} - i_{p2} = \frac{N_s}{N_p}\left(-i_{s2} - i_{s3} + i_{s5} + i_{s6}\right) \end{aligned} \qquad (22.65)$$

The transformer primary currents and the line currents are

$$i_p = \frac{1}{\sqrt{3}} I_o \qquad i_L = \sqrt{\frac{2}{3}}\, I_o \qquad (22.66)$$

The transformer power ratings (which are relatively poor) are

$$\begin{aligned} S_s &= 6\left(\frac{\pi}{3\sqrt{2}} V_o\right)\frac{I_o}{\sqrt{6}} = \frac{\pi}{\sqrt{3}} P_o = 1.81 P_o \\ S_p &= 3\left(\frac{\pi}{3\sqrt{2}} V_o\right)\frac{I_o}{\sqrt{3}} = \frac{\pi}{\sqrt{6}} P_o = 1.28 P_o \\ \overline{S} &= ½\left(\frac{\pi}{\sqrt{3}} P_o + \frac{\pi}{\sqrt{6}} P_o\right) = ½ \frac{\pi}{\sqrt{3}}\left(1 + \frac{1}{\sqrt{2}}\right) P_o = 1.55 P_o \end{aligned} \qquad (22.67)$$

The same primary and secondary apparent powers result for a purely resistive load.

The supply power factor is $pf = 3/\pi = 0.955$.

Independent of the primary connection, the average output voltage is

$$V_o = \frac{3\sqrt{2}}{\pi} V_s \qquad (22.68)$$

and the rms output voltage is

$$V_{o\,rms} = \sqrt{2} V_s \sqrt{\frac{6}{2\pi}\left(\frac{\pi}{6} + \frac{\sqrt{3}}{4}\right)} \qquad (22.69)$$

The diode average and rms currents are

$$I_D = \frac{I_o}{6} \qquad I_{D\,rms} = \frac{I_o}{\sqrt{6}} \qquad (22.70)$$

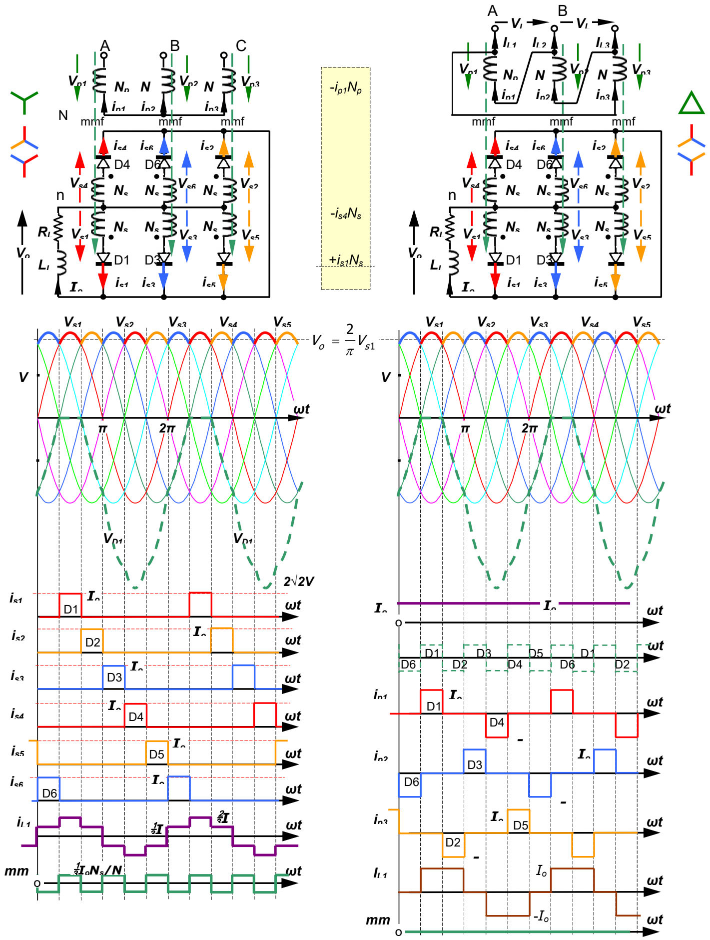
The maximum diode reverse voltage is

$$V_{Dr} = 2\sqrt{2} V_s \qquad (22.71)$$

The line currents are added to the waveforms in figure 22.10a and are also shown in figure 22.11b. The core mmf bias is zero, without any ac component associated with the 6-pulse rectification process. Zero sequence, triplen currents, can flow in the delta primary connection. A star connected primary is therefore not advisable.

If a single-phase inter-wye transformer is used between the neutrals of the two star rectifier groups, the transformer apparent power factors improve significantly, to

$$S_s = 1.48 P_o \quad S_p = 1.05 P_o \quad \text{giving} \quad \overline{S} = 1.26 P_o \qquad (22.72)$$



(a)

Figure 22.10. *Three-phase transformer winding arrangement with hexa-phase rectification:*
(a) star connected primary with dc mmf bias and (b) delta connected primary.
(the transformer secondary and diode currents are the same in each case)

22.1.7 Three-phase transformer mmf imbalance cancellation – zig-zag winding

In figures 22.11a and 22.12a, for balanced input currents and equal turns number N_s in the six windings

$$N_s\left(I_{aa'}+I_{nc'}\right)=N_s\left(I_{an}-I_{cn}\right)$$

whence (22.73)

$$N_s\left(I_{an}-I_{cn}\right)=\sqrt{3}\,N_s I_{an}\angle-30^\circ$$

If the same windings were connected in series in a Y configuration the mmf would be $2NI_{an}$. Therefore 1.15 times more turns ($2/\sqrt{3}$) are needed with the zig-zag arrangement in order to produce the same mmf.

Similarly for the output voltage, when compared to the same windings used in series in a Y secondary configuration:

$$\begin{aligned}V_{na}&=V_{na'}+V_{a'a}\\&=-V_{a'n}+V_{a'a}\\&=\sqrt{3}\,V_{a'a}\angle30^\circ\end{aligned}\qquad(22.74)$$

That is, for a given line to neutral voltage, 1.15 times as many turns are needed as when Y connected.

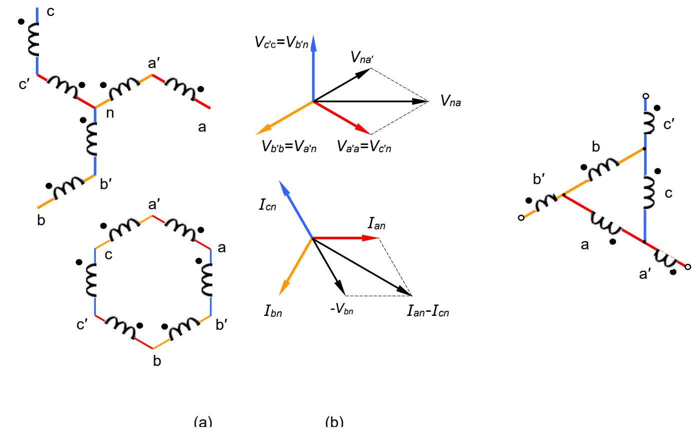


Figure 22.11. *Three-phase transformer secondary zig-zag winding arrangement:*
(a) secondary windings,(b) current and voltage phasors for the fork case, and (c) minimal connections arrangement.

i. star connected primary Y-z (WYE-zigzag)

In figure 22.12, each limb of the core has an extra secondary winding, of the same number of secondary turns, N_s.
MMF analysis of each of the three limbs yields

$$\begin{aligned}&\text{limb 1:-}\quad mmf_o=-i_{p1}N_p+i_{s1}N_s-i_{s3}N_s\\&\text{limb 2:-}\quad mmf_o=-i_{p2}N_p+i_{s2}N_s-i_{s1}N_s\\&\text{limb 3:-}\quad mmf_o=-i_{p2}N_p+i_{s3}N_s-i_{s2}N_s\end{aligned}\qquad(22.75)$$

$$i_{p1}+i_{p2}+i_{p3}=0$$

Adding the three *mmf* equations gives $mmf_o = 0$ and the alternating primary (and line) currents are

$$i_{p1}=\frac{N_s}{N_p}\left(i_{s1}-i_{s3}\right)\qquad i_{p2}=\frac{N_s}{N_p}\left(i_{s2}-i_{s1}\right)\qquad i_{p3}=\frac{N_s}{N_p}\left(i_{s3}-i_{s2}\right)\qquad(22.76)$$

These equations are plotted in figure 22.12a.

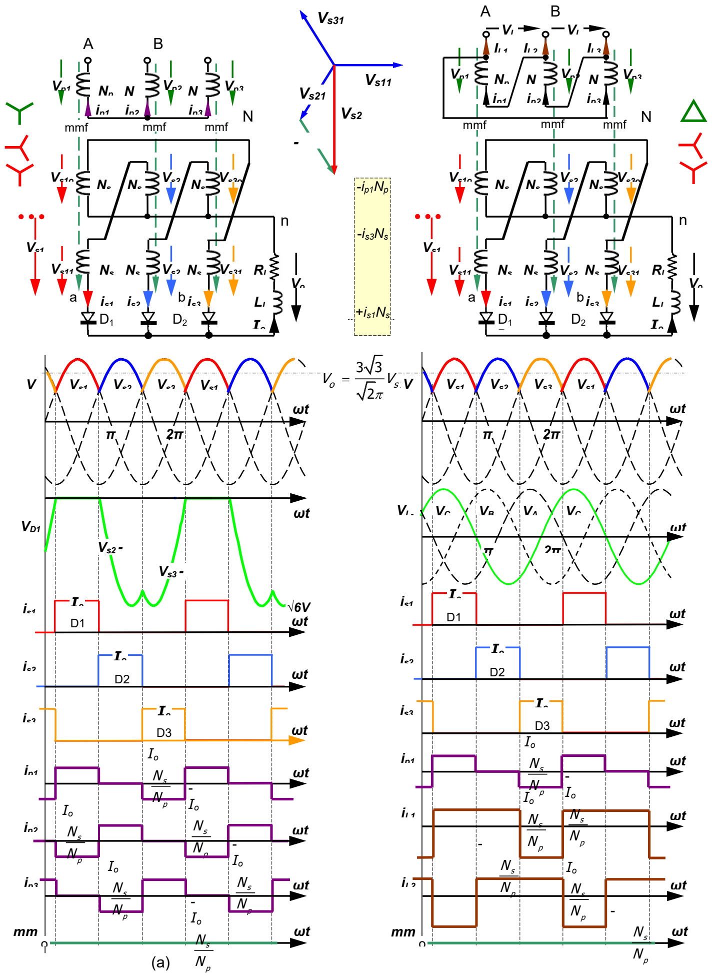


Figure 22.12. *Three-phase transformer winding zig-zag arrangement with no dc mmf bias: (a) star connected primary and (b) delta connected primary.*

If a 1:1:1 turns ratio is assumed, the power ratings of the transformer (which is independent of the turns ratio) involves the vectorial addition of the winding voltages.

$$\vec{V}_{s1} = \vec{V}_{s11} - \vec{V}_{s2o}$$
$$\vec{V}_{s2} = \vec{V}_{s21} - \vec{V}_{s3o} \quad (22.77)$$
$$\vec{V}_{s3} = \vec{V}_{s31} - \vec{V}_{s1o}$$

The various transformer ratings are

$$S_s = 3I_sV_{s0} + 3I_sV_{s1} = 6I_sV_{so} = \frac{2\sqrt{2}\pi}{3\sqrt{3}}P_o \qquad S_p = 3I_pV_p = \frac{2\pi}{3\sqrt{3}}P_o$$
$$\bar{S} = \tfrac{1}{2}\left(S_s + S_p\right) = P_o\frac{\pi}{3\sqrt{3}}\left(\sqrt{2}+1\right) = 1.46P_o \quad (22.78)$$

ii. delta connected primary Δ-z (DELTA-zigzag)

Carrying out an *mmf* balancing exercise, assuming no alternating *mmf* component, and the mean line current is zero, yields

$$i_{L1} = i_{p1} - i_{p3} = \frac{N_s}{N_p}\left(i_{s1} + i_{s2} - 2i_{s3}\right)$$
$$i_{L2} = i_{p2} - i_{p1} = \frac{N_s}{N_p}\left(i_{s2} + i_{s3} - 2i_{s1}\right) \quad (22.79)$$
$$i_{L3} = i_{p3} - i_{p2} = \frac{N_s}{N_p}\left(i_{s3} + i_{s1} - 2i_{s2}\right)$$

The primary and secondary currents are the same whether for a delta or star connected primary, therefore

$$\bar{S} = \tfrac{1}{2}\left(S_s + S_p\right) = 1.46P_o \quad (22.80)$$

If a 1:1:1 turns ratio is assumed, the line, primary and load current are related according to

$$I_L = \sqrt{\frac{2}{3}I_o^2 + \frac{4}{3}I_o^2} = \sqrt{2}I_o$$
$$I_p = \sqrt{\frac{2}{3}}I_o \qquad I_L = \sqrt{3}I_p \quad (22.81)$$

A zig-zag secondary can be a Y-type fork for a possible neutral connection or alternatively, a Δ-type polygon when the neutral is not required.

Each diode conducts for 120º and

$$\bar{I}_D = \tfrac{1}{3}I_o \qquad I_{D\,rms} = \frac{I_o}{\sqrt{3}} \qquad V_D = \sqrt{6}\frac{N_s}{N_p}V_p \quad (22.82)$$

22.1.8 Three-phase transformer full-wave rectifiers – zero core mmf

Full-wave rectification is common in single and three phase applications, since, unlike half-wave rectification, the core *mmf* bias tends to be zero. In three-phase, it is advisable that either the primary or secondary be a delta connection. Any non-linearity in the core characteristics, namely hysteresis, causes triplen fluxes. If a delta connection is used, triplen currents can circulate in the winding, thereby suppressing the creation of triplen core fluxes. If a Y-y connection is used, a third winding set, delta connected, is usually added to the transformer in high power applications. The extra winding can be used for auxiliary type supply applications, and in the limit only one turn per phase need be employed if the sole function of the tertiary delta winding is to suppress core flux triplens.
The primary current harmonic content is the same for a given output winding configuration, independent of whether the primary is star or delta connected.

i. Star connected primary Y-y (Wye-wye)

The Y-y connection shown in figure 22.13a (with primary and secondary neutral nodes *N*, *n* respectively) is the simplest to analyse since each phase primary current is equal to a corresponding phase secondary current.

$$\begin{aligned} mmf_o &= N_s i_{s1} - N_p i_{p1} \\ mmf_o &= N_s i_{s2} - N_p i_{p2} \\ mmf_o &= N_s i_{s2} - N_p i_{p2} \end{aligned} \quad (22.83)$$

Adding the three *mmf* equations gives

$$3 \times mmf_o = N_p \sum_{i=1}^{3} i_{pi} - N_s \sum_{i=1}^{3} i_{si} \quad (22.84)$$

but

$$i_{p1} + i_{p2} + i_{p3} = 0 \quad (22.85)$$

and the secondary currents always sum to zero, then $mmf_o = 0$.

Additionally

$$i_{L1} = i_{p1} = \frac{N_s}{N_p} i_{s1} \qquad i_{L2} = i_{p2} = \frac{N_s}{N_p} i_{s2} \qquad i_{L3} = i_{p3} = \frac{N_s}{N_p} i_{s3} \quad (22.86)$$

Generally

$$I_p = \frac{N_s}{N_p} I_s = \frac{N_s}{N_p} \sqrt{\frac{2}{3}} I_o \quad (22.87)$$

whence

$$\begin{aligned} S_p &= \frac{2\pi}{3\sqrt{3}} P_o = 1.21 P_o \qquad S_s = \frac{\sqrt{2}\pi}{3} P_o = 1.48 P_o \\ \overline{S} &= ½\left(\frac{2\pi}{3\sqrt{3}} P_o + \frac{\sqrt{2}\pi}{3} P_o\right) = 1.35 P_o \end{aligned} \quad (22.88)$$

The secondary harmonic currents are given by

$$I_{sh} = \frac{1}{h} I_{s1} = \frac{1}{h} \frac{\sqrt{6}}{\pi} I_o \quad \text{for } h = 6n \pm 1 \quad \forall \quad n > 0 \quad (22.89)$$

The full-wave, three-phase rectified average output voltage (assuming the appropriate turns ratio, 1:1, to give the same output voltage for a given input line voltage) is

$$V_o = \frac{3\sqrt{3}}{\pi} V_p = \frac{3}{\pi} V_L \quad (22.90)$$

The fundamental ripple in the output voltage, at six times the supply frequency, is $0.057V_o$.

Since with a star primary the line currents are the primary currents, the supply power factor is

$$pf = \frac{P_o}{S} = \frac{3}{\pi} = 0.955 \quad (22.91)$$

ii. Delta connected primary Δ-y (Delta-wye)

The secondary phase currents in figure 22.13b are the same as for the Y-y connection, but the line currents are composed as follows

$$i_{L1} = i_{p1} - i_{p3} \qquad i_{L2} = i_{p2} - i_{p1} \qquad i_{L3} = i_{p3} - i_{p2} \quad (22.92)$$

Such that

$$\begin{aligned} I_p &= \frac{N_s}{N_p} I_s = \frac{N_s}{N_p} \sqrt{\frac{2}{3}} I_o \\ I_L &= \sqrt{3}\, I_p = \frac{N_s}{N_p} \sqrt{3}\, I_s = \frac{N_s}{N_p} \sqrt{2}\, I_o \end{aligned} \quad (22.93)$$

The secondary harmonic currents are given by

$$I_{sh} = \frac{1}{h} I_{s1} = \frac{1}{h} \frac{\sqrt{6}}{\pi} I_o \quad \text{for } h = 6n \pm 1 \quad \forall \quad n > 0 \quad (22.94)$$

The full-wave, three-phase rectified average output voltage (assuming the appropriate turns ratio, √3:1, to give the same output voltage for a given input line voltage) is

$$V_o = \frac{3\sqrt{3}}{\pi} V_p = \frac{3}{\pi} V_L \quad (22.95)$$

The transformer apparent power components are

$$S_s = 1.05 P_o \quad S_p = 1.05 P_o \quad \text{hence} \quad \overline{S} = 1.05 P_o \quad (22.96)$$

The fundamental ripple in the output voltage, at six times the supply frequency, is $0.057V_o$.

The supply power factor is

$$pf = \frac{3}{\pi} = 0.955 \quad (22.97)$$

iii. Star connected primary Y-δ (Wye-delta)

In the Y-δ configuration in figure 22.14a, there are no zero sequence currents hence no *mmf* bias arises, $mmf_o = 0$, and both transformer sides have positive and negative sequence currents.

$$i_{p1} = \frac{N_s}{N_p} i_{s1} \qquad i_{p2} = \frac{N_s}{N_p} i_{s2} \qquad i_{p3} = \frac{N_s}{N_p} i_{s3} \quad (22.98)$$

and $i_{s1} + i_{s2} + i_{s3} = 0$

where

$$\begin{aligned} i_{L1} &= \frac{N_s}{N_p}\left(i_{s1} - i_{s2}\right) = i_{p1} - i_{p2} \\ i_{L2} &= \frac{N_s}{N_p}\left(i_{s2} - i_{s3}\right) = i_{p2} - i_{p3} \\ i_{L2} &= \frac{N_s}{N_p}\left(i_{s3} - i_{s1}\right) = i_{p3} - i_{p1} \end{aligned} \quad (22.99)$$

Thus the transformer currents are related to the supply line currents by

$$\begin{aligned} i_{p1} &= \frac{N_s}{N_p} i_{s1} = \frac{2}{3} i_{L1} - \frac{2}{3} i_{L2} \\ i_{p2} &= \frac{N_s}{N_p} i_{s2} = \frac{2}{3} i_{L2} - \frac{2}{3} i_{L3} \\ i_{p3} &= \frac{N_s}{N_p} i_{s3} = \frac{2}{3} i_{L3} - \frac{2}{3} i_{L1} \end{aligned} \quad (22.100)$$

where

$$i_{L1} + i_{L2} + i_{L3} = 0 \quad (22.101)$$

Generally

$$I_p = \frac{N_s}{N_p} I_s = \frac{N_s}{N_p} \frac{2\sqrt{2}}{3} ½ I_o \quad (22.102)$$

The full-wave, three-phase rectified average output voltage (assuming the appropriate turns ratio, √3:1, to give the same output voltage for a given input line voltage) is

$$V_o = \frac{3\sqrt{3}}{\pi} V_p = \frac{3}{\pi} V_L \quad (22.103)$$

The fundamental ripple in the output voltage, at six times the supply frequency, is $2/5{\times}7 = 0.057V_o$.

The supply power factor is

$$pf = \frac{3}{\pi} = 0.995 \quad (22.104)$$

for an output power, $P_o = V_o I_o$,

iv. Delta connected primary Δ-δ (Delta-delta)

The phase primary and secondary voltages are in phase.

As shown in figure 22.14b the line currents are composed as follows

$$i_{L1} = i_{p1} - i_{p3} \qquad i_{L2} = i_{p2} - i_{p1} \qquad i_{L3} = i_{p3} - i_{p2} \tag{22.105}$$

The transformer primary and secondary currents are

$$i_{p1} = \frac{N_s}{N_p} i_{s1} \qquad i_{p2} = \frac{N_s}{N_p} i_{s2} \qquad i_{p3} = \frac{N_s}{N_p} i_{s3} \tag{22.106}$$

and

$$\begin{aligned} i_{p1} + i_{p2} + i_{p3} &= 0 \\ i_{s1} + i_{s2} + i_{s3} &= 0 \\ i_{L1} + i_{L2} + i_{L3} &= 0 \end{aligned} \tag{22.107}$$

Generally

$$I_p = \frac{N_s}{N_p} I_s \tag{22.108}$$

The full-wave, three-phase rectified average output voltage (assuming the appropriate turns ratio, 1:1, to give the same output voltage for a given input line voltage) is

$$V_o = \frac{3\sqrt{3}}{\pi} V_p = \frac{3}{\pi} V_L \tag{22.109}$$

The rms output voltage is

$$V_{o\,rms} = \sqrt{2} V_s \sqrt{\frac{3}{2} + \frac{9\sqrt{3}}{4\pi}} \tag{22.110}$$

The fundamental ripple in the output voltage, at six times the supply frequency, is $0.057V_o$.

The primary and secondary apparent powers are

$$S_p = S_s = \frac{\pi}{3} P_o = 1.05 P_o \tag{22.111}$$

Thus the supply power factor is

$$pf = \frac{P_o}{S} = \frac{3}{\pi} = 0.955 \tag{22.112}$$

for an output power, $P_o = V_o I_o$,

In summary, when the primary and secondary winding configurations are the same (Δ-δ or Y-y) the input and output line voltages are in phase, otherwise (Δ-y or Y-δ) the input and output line voltages are shifted by 30º relative to one another.

Independent of the transformer primary and secondary connection, for a specified input and output voltage, the following electrical equations hold.

$$V_o = \frac{3\sqrt{3}}{\pi} V_p = \frac{3}{\pi} \frac{N_s}{N_p} V_L \qquad pf = \frac{3}{\pi}$$

$$\bar{I}_D = \frac{1}{3} I_o \qquad I_{D\,rms} = \frac{1}{\sqrt{3}} I_o \qquad V_{DR} = \sqrt{3}\sqrt{2} V_s$$

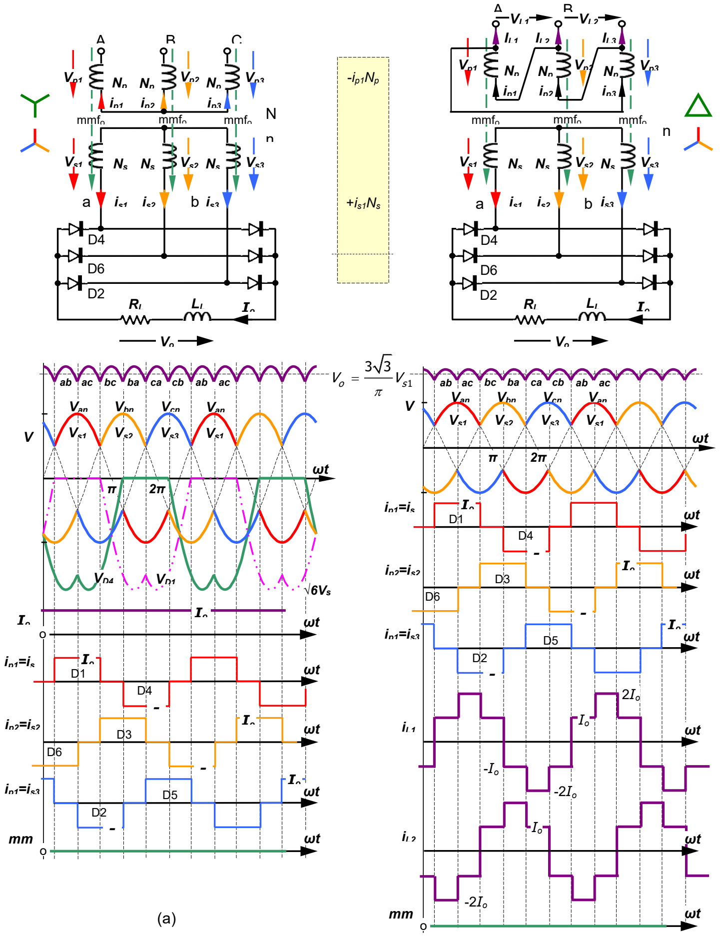


Figure 22.13. *Three-phase transformer wye connected secondary winding with full-wave rectification and no resultant dc mmf bias: (a) star connected primary Y-y and (b) delta connected primary Δ-y.*

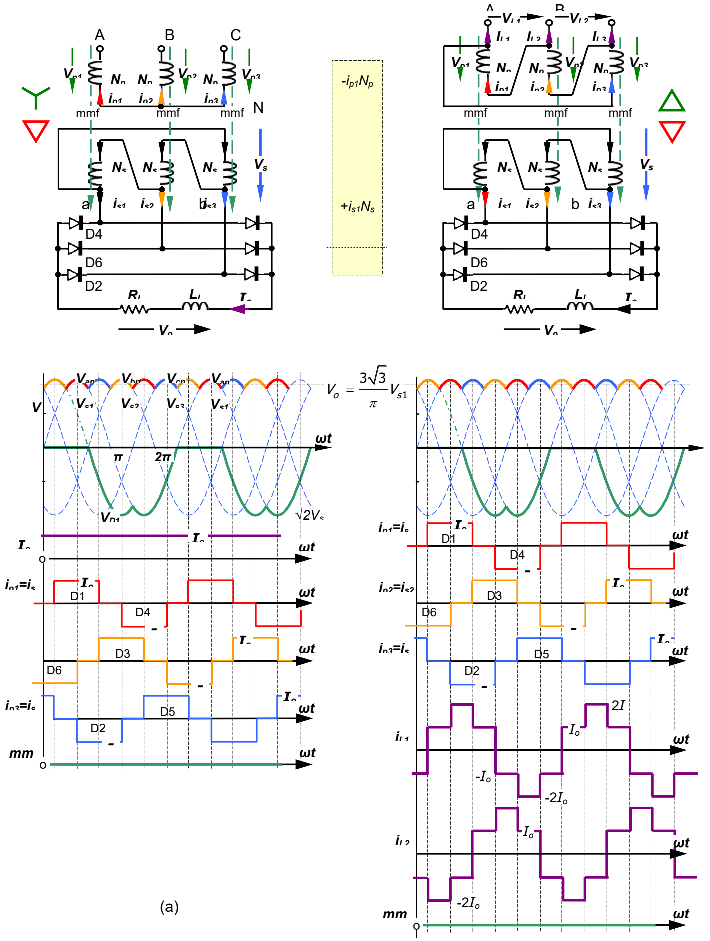


Figure 22.14. *Three-phase transformer with delta connected secondary winding with full-wave rectification and no resultant dc mmf bias: (a) star connected primary Y- δ and (b) delta connected primary Δ- δ.*

22.2 Auto-transformers

Auto-transformer Construction

Auto-transformer construction is characterized by a single copper wire coil common to both the primary and secondary circuits, wound around a silicone steel core. While theoretically separate parts of the winding can be used for input and output, in practice the higher voltage will be connected to the ends of the winding, and the lower voltage from one end to a tap. For example, a transformer with a tap at the centre of the winding can be used with 230 Vac across the entire winding, and 115 Vac equipment, or reversed to drive 230 Vac equipment from 115 Vac. As the same winding is used for input and output, the flux in the core is partially cancelled, and a smaller core can be used. For voltage ratios not exceeding about 3:1, auto transformer is cheaper, lighter, smaller and more efficient than a true (two-winding) transformer of the same rating. By exposing part of the winding coils and making the secondary connection through a sliding brush, an auto transformer with a near-continuously variable turns ratio can be obtained, allowing for very small increments of voltage.

Auto-transformer Operation

An autotransformer is a single winding electrical transformer that has at least three electrical connections called taps as shown in figure 22.15 parts *b* and *c*. The voltage source and the load are each connected to two taps. One tap, normally at the end of the winding is a common connection to both circuits (source and load). Each tap corresponds to a different source or load voltage. In an auto-transformer a portion of the same winding is part of both the primary and secondary winding.

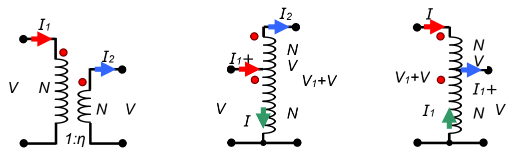


Figure 22.15. *Transformer and autotransformer connection diagram: (a) two winding transformer, (b) step-up voltage autotransformer, and (c) step-down voltage autotransformer.*

When the primary side of a transformer with a winding of N_1 turns is supplied with V_1 voltage (satisfying $V = Nd\varphi/dt$), then on the secondary N_2 turns, is induced V_2 voltage, in accordance with the *transformer turns ratio* η_T:

$$\eta_T = \frac{V_2}{V_1} = \frac{N_2}{N_1} = \frac{I_1}{I_2} \qquad (22.113)$$

In the transformer, the power is transmitted from the primary to the secondary circuit via the magnetic field, such that the input VA is equal to the output VA, assuming lossless windings and neglecting any magnetising current. That is

$$S = VA_{Xfm} = V_1 I_1 = V_2 I_2 \qquad (22.114)$$

The energy flow through the autotransformer is the summation of magnetic transformation (induction) phenomena and input to output current conduction. The conduction results from the series connection of the autotransformer's primary and secondary circuits. The accumulative (additive) series connection is assumed, as indicated by the dots in figure 22.15, since no advantages are gain when the windings are differentially connected.

The VA capability of an auto-transformer does not depend on whether the common connection point forms an input terminal or output terminal. It is assumed that each winding exploits its full VA rating, as define by equation (22.114). When the common connection forms the auto transformer input, giving a step-up voltage, as in figure 22.15b, the voltage transfer ratio is $V_{out} / V_{in} = (V_1 + V_2) / V_1 = 1 + \eta_T$ and the input (and output) VA is:

$$
\begin{aligned}
VA_{autoX-i/p} &= V_1 I_{in} = V_1\left(I_1+I_2\right)\\
&= V_1\left(I_1+\frac{I_1}{\eta_T}\right) = V_1 I_1\left(1+\frac{1}{\eta_T}\right)\\
&= VA_{Xfm}\left(1+\frac{1}{\eta_T}\right)
\end{aligned}
$$

When the common connection forms the auto transformer output, giving a step-down output voltage, as in figure 22.15c, the voltage transfer ratio is $V_{out}\,/\,V_{in} = V_1\,/\,(V_1 + V_2\,) = 1\,/\,(1 + \eta_T)$ and the output (and input) VA is:

$$
\begin{aligned}
VA_{autoX-o/p} &= V_1 I_{out} = V_1\left(I_1+I_2\right)\\
&= V_1\left(I_1+\frac{I_1}{\eta_T}\right) = V_1 I_1\left(1+\frac{1}{\eta_T}\right)\\
&= VA_{Xfm}\left(1+\frac{1}{\eta_T}\right)
\end{aligned}
$$

In each case, the first term is associated with transformer action between the two windings, while the second term is that current component that conducts from the input to the output.
Generally

$$VA_{autoX} = VA_{Xfm}\left(1\pm\frac{1}{\eta_T}\right) \qquad (22.115)$$

where the plus sign is applicable to when the two windings are additively connected (as in figure 22.15b and 22.15c), while the negative sign implies the two windings are connected to oppose. Opposing windings offer poor copper utilisation.

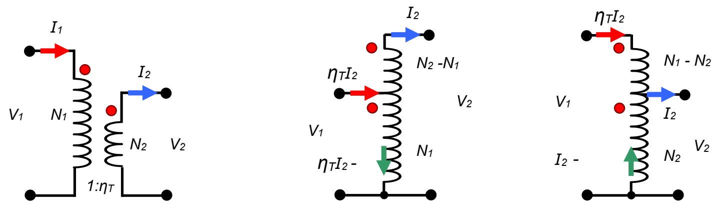


Figure 22.16. *Transformer and autotransformer diagram for V_1 and V_2 input and output voltages: (a) two winding transformer, (b) step-up voltage autotransformer, and (c) step-down voltage autotransformer.*

The area, hence volume, thence weight, of copper required in a winding is proportional to the number of turns and to the cross sectional area of the wire. In turn the area is proportional to the current to be carried, that is, volume of copper is proportional to *NI*.

Volume of copper α length of the wire × cross sectional area of copper wire
α *N* × *I*

The magnetic circuit is assumed to be identical, satisfying $V=Nd\varphi/dt$. To quantify the copper saving, the total quantity of copper used in an auto-transformer is expressed as a fraction of that used in a two winding transformer, both with the same output VA, V_2I_2. The copper area in the two winding transformer in figure 22.16a is

$$\text{copper in two winding transformer} = k\left(N_1 I_1 + N_2 I_2\right) = 2kN_1 I_1$$

For the step-up autotransformer, shown in figure 22.16b, with the same output VA rating (same input voltage V_1 and same output voltage and current V_2, I_2) as the two winding transformer in figure 2a:

$$\frac{\text{copper in auto-transformer}}{\text{copper in two winding transformer}} = \frac{N_1\left(\eta_T I_2 - I_2\right)+\left(N_2-N_1\right)I_2}{N_1 I_1 + N_2 I_2} = \frac{2N_2 I_2\left(1-\dfrac{1}{\eta_T}\right)}{2N_2 I_2} = 1-\frac{1}{\eta_T}$$

The pu copper saving for the step-up autotransformer is $1/\eta_T$, where $\eta_T \geq 1$.

For the step-down autotransformer in figure 22.16c:

$$\frac{\text{copper in auto-transformer}}{\text{copper in two winding transformer}} = \frac{N_2\left(I_2 - \eta_T I_2\right)+\left(N_1-N_2\right)\eta_T I_2}{N_1 I_1 + N_2 I_2} = \frac{2N_2 I_2\left(1-\eta_T\right)}{2N_2 I_2} = 1-\eta_T$$

The pu copper saving for the step-down autotransformer is η_T, where $\eta_T \leq 1$.

Generally, for cumulatively connected autotransformer windings:

$$\frac{\text{copper in auto-transformer}}{\text{copper in two winding transformer}} = 1-\frac{1}{\eta_T^{\pm 1}} \qquad (22.116)$$

Generally the copper saving is $1/\eta_T^{\pm 1}$ where the positive sign is applicable to a voltage step-up connection, while the negative sign implies voltage step-down (with cumulative windings, as opposed to subtractive connection, in each case).

The current in the common part of the autotransformer winding is small compared with input and output currents, being the difference between the two currents. Thus, the cross-section of this part of the winding may be decreased, resulting considerable savings. Although the iron area is unchanged, since the voltages are unchanged ($V = Nd\varphi/dt = kNBAf$), the core window area, hence core length, can be decreased because of the copper area saving. Using smaller quantities of iron and copper results in lower losses and increased efficiency.

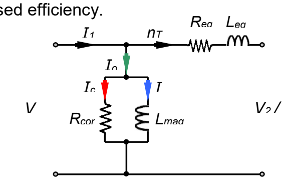


Figure 22.17. *Autotransformer equivalent circuit.*

Equivalent circuit

In figure 22.17, auto-transformer output side resistance R_2 and reactance X_2 transfer to the input according to:

$$
\begin{aligned}
R_{eq\,autoX} &= R_1 + \left(\frac{1}{\eta_T}-1\right)^2 R_2\\
X_{eq\,autoX} &= X_1 + \left(\frac{1}{\eta_T}-1\right)^2 X_2
\end{aligned}
\qquad (22.117)
$$

The two winding transformer equivalent equations are

$$
\begin{aligned}
R_{eq\,Xfm} &= R_1 + \frac{1}{\eta_T^2} R_2\\
X_{eq\,Xfm} &= X_1 + \frac{1}{\eta_T^2} X_2
\end{aligned}
\qquad (22.118)
$$

These equations show that the impedance transferred for the autotransformer is less than for the conventional two winding transformer. Thus in the case of an autotransformer, the short circuit impedance is lower. Having a smaller value of short circuit impedance is considered a disadvantage, since the short circuit currents are larger. But the full load regulation is lower (better).

The autotransformer short circuit voltage compared to a transformer is:

$$\frac{V_{Z\,autoX}}{V_{Z\,Xfm}} = \left(1-\frac{1}{\eta_T}\right)$$

where: $V_{Z\,autoX}$, $V_{Z\,Xfm}$ are auto-transformer and transformer short circuit voltages respectively.

A second autotransformers disadvantage concerns the galvanic (electrical non-isolated) connection of the primary and secondary circuits, due to which, all disturbances, over-voltages, etc. are transmitted directly through conduction between the input and output sides.

Advantages of the auto-transformer

- Using an autotransformer connection offers lower series impedances and better regulation. Its efficiency is more when compared with the conventional transformer.
- In conventional transformer the voltage step up or step down value is fixed while in autotransformer, we can vary the output voltage as per out requirements and can smoothly increase or decrease its value as per our requirement.
- A saving in winding material (less copper or aluminium), since the secondary winding is part of the primary. Smaller volume, hence lower weight.
- Lower copper loss, lower I^2R losses, therefore efficiency is higher than in the two winding transformer.
- Lower leakage reactances, lower magnetising current.
- Variable output voltage obtainable.
- Lower % (better) voltage regulation.
- Low requirements of excitation current.
- There are decreased losses for a given kVA capacity.
- Lower cost

Disadvantages of the auto-transformer

- There is a direct electrical connection between the primary and secondary sides. No electrical isolation.
- Should an open-circuit develop across the common winding portion, the full supply voltage is applied to the secondary.
- The short-circuit current is much larger can result from a lower series impedance than for the normal two-winding transformer.
- The autotransformer connection is not possible with certain three-phase connections (Δ-Y and Y- Δ).
- Short circuits can impress voltages significantly higher than operating voltages across the windings of an autotransformer.
- For the same voltage surge at the line terminals, the impressed and induced voltages are greater for an autotransformer than for a two-winding transformer.
- An autotransformer consists of a single winding around an iron core, which creates a change in voltage from one end to the other. Therefore the self-inductance of the winding around the core changes the voltage potential, but there is no isolation of the high and low voltage ends of the winding. So any noise or other voltage anomaly from one side is passed through to the other side. Thus autotransformers are typically only used where there is already filtering or conditioning ahead of it, as in electronic applications, or the downstream device is unaffected by any anomalies, such as an ac motor during starting.

Autotransformers are used in electromagnetic systems for connecting networks with different voltage levels, in start-up systems for large squirrel-cage induction motors and synchronous motors, and where primary and secondary circuits with galvanic non-isolation is permissible, and where lower weight and losses out weigh the expenditure associated with limiting the short circuit current.

The Korndorfer system presented in chapter 15.4.6ii, is a frequently applied solution when starting up asynchronous motors. The start-up takes place in two stages without voltage-free interruptions.

Used in HV Substations due to following reasons:

1. With a normal transformer the size of transformer will be high which leads to heavy weight, more copper and high cost.
2. The tertiary winding used in an autotransformer balances single phase unbalanced loads connected to secondary and it does not pass on these unbalanced currents through to the primary side. Hence harmonics and voltage unbalance are eliminated.
3. Tertiary winding in the autotransformer balances the amp turns so that the autotransformer achieves magnetic separation like a two winding transformer.

A variable autotransformer is known as a variac (which is a trade mark of General Radio). A variac is a single coil with a sweeping arm for the tap-off, which allows the ratio of primary turns to the secondary turns to be readily altered.

Converter transformer for HVDC bipolar transmission system ± 800kVdc, 6,400MW; 2,071km: single-phase; 550kVac, 816kVdc; 321MVA; high pulse wye system feeding.

Converter transformer for HVDC bipolar transmission system ± 500kVdc; 2,500MW: single-phase; 420kVac; 515kVdc; 397MVA; wye system (on left) and delta system (on right).

22.3 Types of Transformers

Type of insulation transformer: dry or liquid filled

Liquid-filled transformers, most often used by electric utilities, have several efficiency performance advantages over dry-type transformers. Liquid-filled transformers tend to be more efficient (reaching 99.5% efficient – see figures 22.18 and 22.19), have greater overload capability and a longer service life (due to a greater ability to reduce hot-spot coil temperatures and higher dielectric withstand ratings). Liquid-filled transformers are also physically smaller than dry-types for a given kVA rating, which can be important in areas with restricted space, such as offshore applications. However, liquid-filled transformers are often filled with mineral oil which has a higher flammability potential than dry-types and local environmental laws may require containment troughs to guard against insulating fluid leaks.

Using liquid (usually mineral oil) as both an insulating and cooling medium, liquid-filled transformers incorporate spacers between the windings to allow the fluid to flow and cool the windings and core. The heat removed from the core-coil assembly by the fluid is then exhausted to the environment through the tank walls (which can include fins to enhance cooling effectiveness), or through the use of external radiators with passive or active fluid circulation and cooling fans. The standard winding insulation used in liquid-filled units consists of thermally upgraded nitrogen-rich Kraft paper, mineral oil and magnet wire covered with enamel or thermally upgraded Kraft paper that anticipates a 20°C average ambient for its expected life. However the thermal index for such a system is 180,000 hours (20 years continuous operation) at 110°C, consisting of 65°C average winding rise, 15°C hot spot increment and a maximum average ambient of 30°C for 24 hours or maximum peak ambient of 40°C. At higher ambient temperatures, the winding temperature rise has to be reduced to operate within the same 110°C hot spot.

Dry-type transformers tend to be used most often by commercial and industrial customers. Generally, higher-capacity transformers used outdoors are almost always liquid-filled, while lower-capacity transformers used indoors are often dry-type. Dry-type transformers typically are housed in enclosures, with the windings insulated through varnish, vacuum pressure impregnated varnish, epoxy resin or cast resin. Dry type insulation can provide excellent dielectric strength and are often designed to withstand temperatures up to 220°C. Temperature rise ratings of dry-type transformers are based on the thermal performance of the type of insulation used - three ratings are 220°C, 185°C, and 150°C, each with a 30°C hot spot allowance. A 105°C insulation class with a 10°C hot spot increment is generally reserved for fractional kVA transformers.

Table 22.1 summarises the broad types of transformers and describes their most common uses. While the naming conventions are not necessarily universally consistent, from a practical perspective, the table does represent how the transformer types are used in transmission and distribution systems.

Table 22.1. General Transformer Types

Transformer type	Voltage	Phases	Typical Insulation	Common Use
Large Power	>245 kV (High voltage)	Single and Three	Liquid-filled	Stepping up to or down from higher voltages for electricity transmission over distances; substation transformers
Medium Power	>36kV and ≤230kV (Medium voltage)	Single and Three	Dry-type or liquid-filled	Stepping voltages down from a sub transmission system to a primary distribution system
Medium Voltage Distribution	≤36 kV (Medium voltage)	Single and Three	Dry-type or liquid-filled	Stepping voltages down within a distribution circuit from a primary to a secondary distribution voltage
Low Voltage Distribution	≤1 kV (Low voltage)	Single and Three	Dry-type	Stepping voltages down within a distribution circuit of a building or to supply power to equipment

Distribution transformers can have various types of cores with laminated steel being the most common. Using amorphous metal as the core material over laminated steel can reduce the core losses by approximately 60%. Amorphous alloys are produced by a process of rapidly cooling a liquid metal in order for it to keep its liquid, non-crystalline structure. The advantage of an amorphous alloy is its strength and electrical characteristics. A disadvantage is that it requires advanced, more-costly techniques to produce.

Transformers made with amorphous cores have advantages such as low core losses, less noise, higher efficiency and a longer life. Their disadvantages include higher inrush current, more harmonic problems, larger size and higher initial cost.

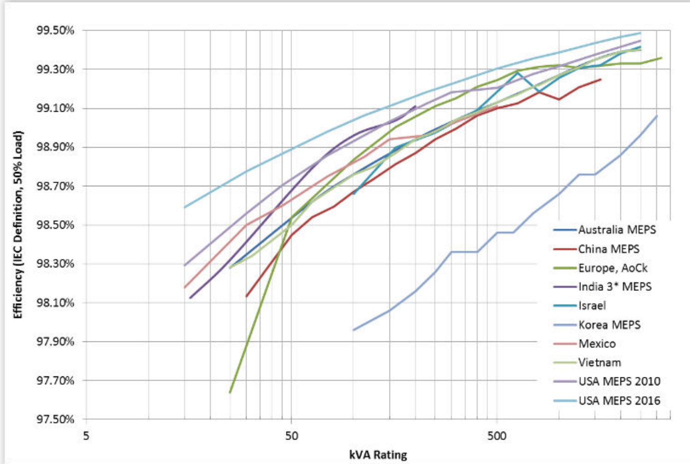


Figure 22.18. *Efficiency at 50% load, 50Hz, based on power input for three-phase liquid-filled transformers (log scale).*

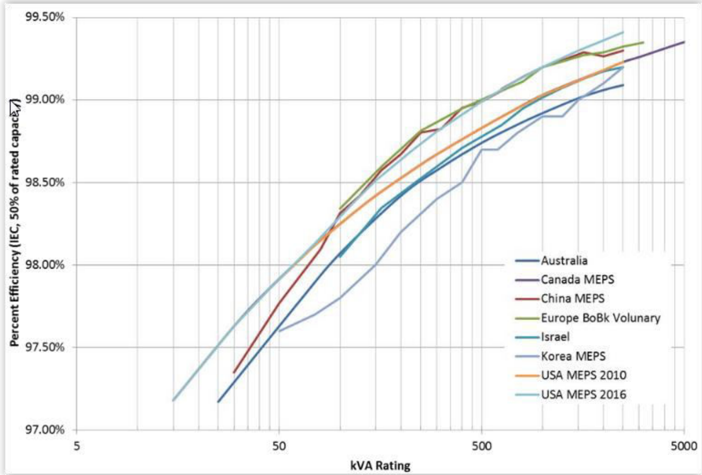


Figure 22.19. *Efficiency at 50% load, 50Hz, based on power input, for three-phase dry-type distribution transformers (log scale).*